PROJECTION OF A SIDE OF A TRIANGLE

In this unit, students will learn how to:

Prove the following theorems along with corollaries and apply them to solve appropriate problems.

- In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle .contained by one of the sides, and the projection on u of the other.
- In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.
- In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apollonius' Theorem).

THEOREM 1

8.1 (i) In an obtuse angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares, on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.

Given:

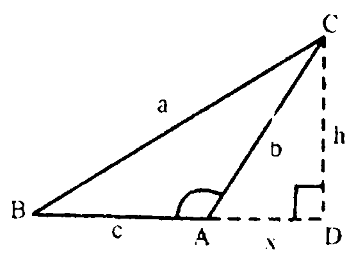
ABC is a triangle having an obtuse angle

BAC at A. Draw $\overline{CD}$ perpendicular on $\overline{BA}$ produced.

So that $\overline{AD}$ is the projection of $\overline{AC}$ on $\overline{BA}$ produced.

Take $m\overline{BC} = a$, $m\overline{CA} = b$, $m\overline{AB} = c$,

$m\overline{AD} = x$ and $m\overline{CD} = h$.



To prove

$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$

i. e., $a^2 = b^2 + c^2 + 2cx$

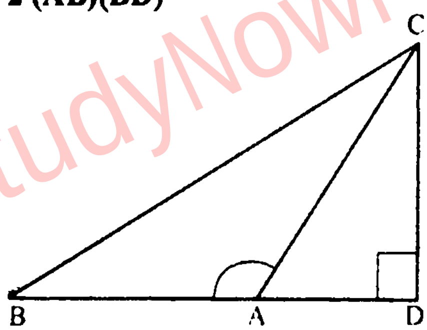
Proof:

Statements	Reasons
In ∠ rt Δ CDA,	
$m\angle CDA = 90°$	Given
∴ $(\overline{AC})^2 = (AD)^2 + (CD)^2$	Pythagoras Theorem
or $b^2 = x^2 + h^2$ ______ (i)	
In ∠ rt Δ CDB,	
$mCDB = 90°$	Given
∴ $(\overline{BC})^2 = (BD)^2 + (CD)^2$	Pythagoras Theorem
or $a^2 = (c + x)^2 + h^2$	$\overline{BD} = \overline{BA} + \overline{AD}$
$= c^2 + 2cx + x^2 + h^2$ (ii)	
Hence, $a^2 = c^2 + 2cx + b^2$	Using (i) and (ii)
i,e., $a^2 = b^2 + c^2 + 2cx$	
or $(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	

Example

In a ΔABC with obtuse angle at A, if $\overline{CD}$ is an altitude on BA produced and $m\overline{AC} = m\overline{AB}$.

Then prove that $(BC)^2 = 2(AB)(BD)$



Given:

In a ΔABC, m∠A is obtuse $m\overline{AC}$ $m\overline{AB}$ and CD being altitude on $\overline{BA}$ produced.

To prove:

$(\overline{BC})^2 = 2(m\overline{AB})(m\overline{BD})$

Proof:

In a ΔABC, having obtuse angle BAC at A.

Statements	Reasons
$(\overline{BC})^2 = (\overline{BA})^2 + (\overline{AC})^2 + 2(m\overline{BA})(m\overline{AD})$	By Theorem 1
$= (\overline{AB})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	Given
$= 2(\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	
$(\overline{BC})^2 = 2m\overline{AB}\,(m\overline{AB} + m\overline{AD})$	On the line segment $\overline{BD}$,
$= 2m\overline{AB}\,(m\overline{AB} + m\overline{AD})$	Point A is between B and D

SOLVED EXERCISE 8.1

Q1. Given $m\overline{AC}$ = 1cm, $m\overline{BC}$ = 2cm, $m\angle C$ = 120°. Compute the length AB and the area of ΔABC.

Hint: $(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 + 2\, mAC.\, m\overline{CD}$

Where $(m\overline{CD}) = (m\overline{BC})\cos(180° - C)$ *(Use theorem 1).*

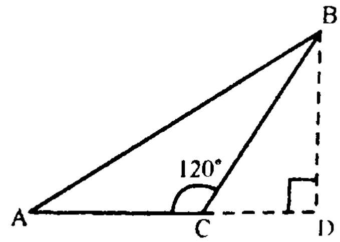
Solution:

Given

$m\overline{AC} = 1cm;\ m\overline{BC} = 2cm;\ m\angle C = 120°$

Required: $m\overline{AB} = ?$

and Area of ABC = ?

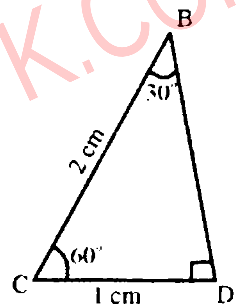


$$m\overline{AB}^2 = m\overline{AC}^2 + m\overline{BC}^2 + 2m\overline{AC}\,m\overline{CD}$$

$$= (1)^2 + (2)^2 + 2(1)(\overline{CD})$$

$$= 1 + 4 + 2\overline{CD}$$

$$= 1 + 4 + 2\overline{CD} \quad \_\_\_\_\_\_\_\_ (i)$$



In Δ BCD

$m\angle BCD = 60°$

and $m\angle CBD = 30°$

The side opposite to ∠ 30° is $\overline{CD}$ which is $\frac{1}{2}\overline{CB}$, the hypotenuse of right Δ CDB.

$\overline{CD} = 1cm$

By putting the value of $\overline{CD}$ in eq. (i)

$$m\overline{AB}^2 = 5 + 2(1)(1) = 5 + 2 = 7,$$

$$m\overline{AB}^2 = 7 \Rightarrow m\overline{AB} = \sqrt{7}cm = 2.646cm$$

And $m\overline{CB}^2 = m\overline{CD}^2 + \overline{BD}^2$

$$2^2 = 1^2 + m\overline{BD}^2$$

$$4 = 1 + m\overline{BD}^2$$

$$mBD^2 = 4 - 1 = 3$$

$$h = m\overline{BD} = \sqrt{3}$$

Area of ABC.

$$= \frac{1}{2}\text{ base} \times \text{height}$$

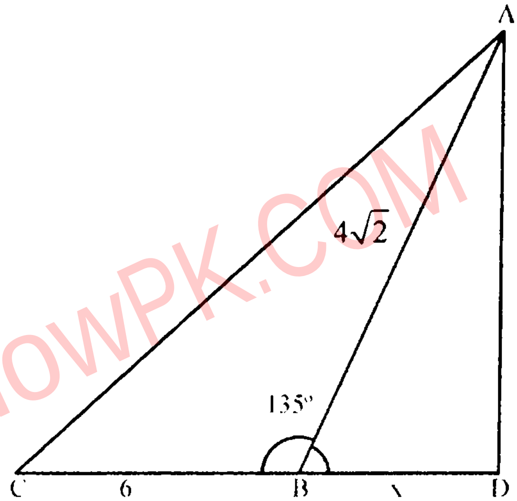
$$= \frac{1}{2} \text{ m } \overline{AC} \times \text{m } \overline{BD}$$

$$= \frac{1}{2} \times 1 \times \sqrt{3}$$

$$\text{Area of ABC} = \frac{\sqrt{3}}{2} \text{cm}$$

Q2. Find m $\overline{AC}$ if m$\overline{CB}$ = 6 cm, $\overline{CB}$ = 6 cm, m $\overline{AB}$ = $4\sqrt{2}$ cm and m∠ABC = 135°.

Solution:



Let m $\overline{BD}$ = x

In Δ ABD, we have

$$\text{Cos } 45^\circ = \frac{\overline{BD}}{\overline{AB}}$$

$$\frac{1}{\sqrt{2}} = \frac{x}{4\sqrt{2}}$$

$$\sqrt{2}\ x = 4\sqrt{2}$$

$$x = 4 \text{ cm}$$

we know that

To Prove:

$(m\overline{AC})^2 = (m\overline{CB})^2 + (m\overline{AB})^2 + 2 \times m\overline{CB} \times$ m $\overline{BD}$

$$= (6)^2 + (4\sqrt{2})^2 + 2 \times 6 \times 4$$

$$= 36 + 32 + 48$$

$$= 116$$

$$\Rightarrow \quad m\overline{AC} = \sqrt{116} = \sqrt{4 \times 29} = 2\sqrt{29} \text{ cm}$$

THEOREM 2

8.1 (ii) In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

Given:

ΔABC with an acute angle CAB at A.

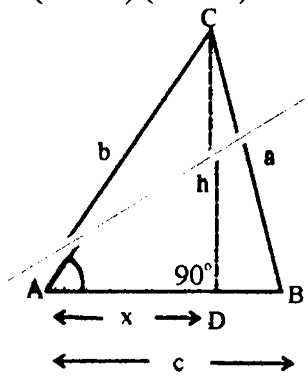
Take m $\overline{BC}$ a, m $\overline{CA}$ = b and mAB = c $\overline{AB}$

Draw $\overline{CD} = \overline{AB}$ so that $\overline{AD}$ is projection of $\overline{AC}$ on $\overline{AB}$

Also, $m\overline{AD} = x$ and $m\overline{CD} = h$

To prove:

$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$ i.e., $a^2 = b^2 + c^2 - 2cx$



Proof:

Statements	Reasons
In $\angle$ rt $\triangle CDA$	
$m\angle CDA = 90°$	Given
$(\overline{AC})^2 = (\overline{AD})^2 + (\overline{CD})^2$	Pythagoras Theorem
i.e., $b^2 = x^2 + h^2$ (i)	
In $\angle$rt $\triangle CDB$,	
$m\angle CDB = 90°$	Given
$(\overline{BC})^2 = (\overline{AD})^2 + (\overline{CD})^2$	Pythagoras Theorem
$a^2 = (c - x)^2 + h^2$	From the figure
or $a^2 = c^2 - 2cx + x^2 + h^2$ (ii)	
$a^2 = c^2 - 2cx + b^2$	Using (i) and (ii)
Hence, $a^2 = b^2 + c^2 - 2cx$	
i,e., $(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(mAB)(m\overline{AD})$	

THEOREM 3

(Apollonius theorem)

8.1 (iii) In any triangle, the sum of the squares on ? the square on half the third side together median which bisects the third side.

Given

In a ΔABC, the median $\overline{AD}$ bisects $\overline{BC}$ i.e., $m\overline{BD}$ = ?

To prove

$(\overline{AB})^2 + (\overline{AC})^2 = 2(\overline{BD})^2 + 2(\overline{AD})^2$

Construction:

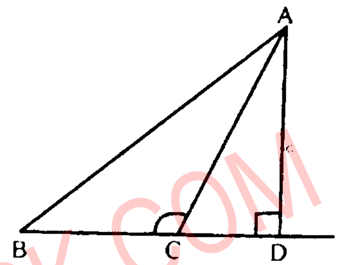
Draw $\overline{AF} \perp \overline{BC}$

Proof:

Statements	Reasons
In ΔADB Since $\angle ADB$ is acute at D	
$\therefore$ $(\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2 - 2\, m\overline{BD} \,.\, m\overline{FD}$ (i)	Using Theorem 2
Now in ΔADC Since $\angle ADC$ is obtuse at D	
$(\overline{AC})^2 = (\overline{CD})^2 + (\overline{AD})^2 + 2\, m\overline{CD} \,.\, m\overline{FD}$	Using Theorem 1
$= (\overline{BD})^2 + (\overline{AD})^2 + 2\, m\overline{BD} \,.\, m\overline{FD}$ (ii)	
Thus $(\overline{AB})^2 + (\overline{AC})^2 = 2(\overline{BD})^2 + 2(\overline{AD})^2$	Adding (i) and (ii)

Example 1

In ΔABC, $\angle C$ is obtuse, $\overline{AD} \perp \overline{BC}$ produced, whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$.
Prove that $(\overline{AC})^2 = (\overline{AB})^2 + 2(\overline{BC})^2 - 2\, m\overline{BC} \,.\, m\overline{BD}$



Given:

In a ΔABC, $\angle BCA$ is obtuse so that $\angle B$ is acute, $\overline{AD} \perp \overline{BC}$ produced, whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$ produced.

To prove:

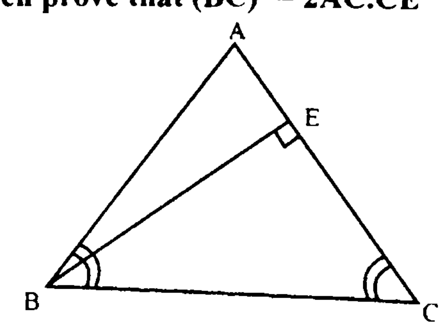
$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\, m\overline{BC} \,.\, m\overline{BD}$

Proof:

Statements	Reasons
In $\angle$ rt ΔAAD	
$(\overline{AB})^2 = (\overline{AD})^2 + (\overline{BD})^2$ (i)	Pythagoras Theorem
In $\angle$ rt ΔACD $(\overline{AC})^2 = (\overline{AD})^2 + (\overline{CD})^2$ (ii)	Pythagoras Theorem
or $(\overline{AC})^2 = (\overline{AD})^2 + (\overline{BD} - \overline{BC})^2$	mBC + mCD = BD
$(\overline{AC})^2 = (\overline{AD})^2 + (\overline{BD})^2 + (\overline{BC})^2 - 2\, m\overline{BC}\ m\overline{BD}$ (iii)	
$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\, m\overline{BC}\ m\overline{BD}$	Using (i) and (iii)

Example 2

In an Isosceles ΔABC, if mAB = mAC and BE $\perp$ AC, then prove that $(BC)^2 = 2AC.CE$



Given

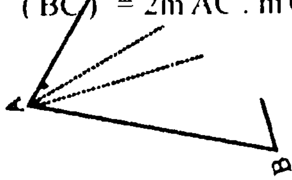
In an Isosceles ΔABC

$m\overline{AB} = m\overline{AC}$ and $\overline{BE} \perp \overline{AC}$

whereas $\overline{CE}$ is the projection of $\overline{BC}$ upon on $\overline{AC}$.

To prove

$(\overline{BC})^2 = 2m\overline{AC} \,.\, m\overline{CE}$



Proof: .

Statements	Reasons
In an isosceles ΔABC with $m\overline{AB} = m\overline{AC}$. If $\angle C$ is acute,	
then $(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2m\overline{AC}.m\overline{CE}$,	By Theorem 2
$(\overline{AC})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2\,m\overline{AC}.m\overline{CE}$	Given mAB = mAC
$\Rightarrow$ $(\overline{BC})^2 - 2m\overline{AC}.m\overline{CE} = 0$	Cancel $(AC)^2$ on bath sides
or $(\overline{BC})^2 = 2m\overline{AC}\ , m\overline{CE}$	

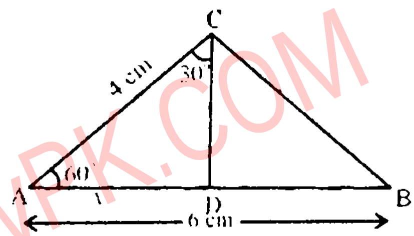
SOLVED EXERCISE 8.2

Q1. In a ΔABC calculate $m\overline{BC}$ when $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$.

Solution:

Given: mAB = 6cm; mAC = 4cm; m CA = 60°.

Required: $m\overline{CB} = ?$



In Δ ABC, we have

$(\overline{BC})^2 = (\overline{AB}) + (\overline{AC})^2 - 2(\overline{AB}).(\overline{AD})$

$= (6)^2 + (4)^2 \times 2(6)(x)$ $\quad \because \cos 60° = \frac{x}{4}$

$= 36 + 16 - 2(6)(2)$ $\quad \frac{1}{2} = \frac{x}{4}$

$= 52 - 24$ $\quad 2x = 4$

$= 28$ $\quad \Rightarrow \quad x = 2$

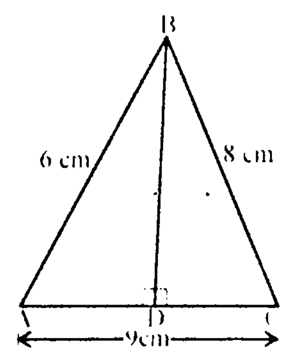
$m\overline{BC} = \sqrt{28}$

$= 2\sqrt{7}$ cm $\Rightarrow$ = 5.29 cm

Q2. In ΔABC, $\overline{AB} = 6$ cm, $\overline{BC} = 8$ cm, $\overline{AC} = 9$ cm and D is the mid point of side $\overline{AC}$. Find length of the median $\overline{BD}$.

Solution:

According to the figure, we have



$m\overline{AD} = \overline{DC}$

and $m\overline{AC} = m\overline{AD} + m\overline{DC}$

$m\overline{AC} = m\overline{AD} + m\overline{AD}$

$9 = 2m\overline{AD}$

Or $2m\overline{AD} = 9$

$m\overline{AD} = \frac{9}{2} = 4.5$ cm

We know that

$$(\overline{AC})^2 + (\overline{BC})^2 = 2\left[(\overline{AD})^2 + (BD)^2\right]$$

$$(6)^2 + (8)^2 = 2\left[(4.5)^2 + (\overline{BD})^2\right]$$

$$36 + 64 = 2(4.5)2 + 2(\overline{BD})^2$$

$$100 = 40.5 + 2\ (\overline{BD})^2$$

Or $$2(BD)^2 = 100 - 40.5$$

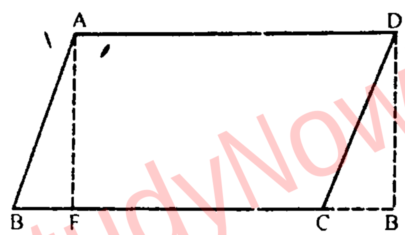
$$2\overline{BD}^2 = 59.5$$

$$\Rightarrow \overline{BD}^2 = 29.75$$

$$\Rightarrow \overline{BD} = \sqrt{29.75} = 5.45 \text{ cm}$$

Q3. In a quadrilateral $\overline{AB}$ CD prove that $(\overline{AC})^2 + (\overline{AD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$

Solution:



$$(\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2\ (\overline{BC})(\overline{CE}) \qquad (1)$$

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\ (\overline{BC})\ (\overline{BF}) \qquad (2)$$

Adding (1) and (2), we get

$$(\overline{AC})^2 + (\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2(\overline{BC})\ \overline{CE} + (\overline{AB})^2 + (\overline{BC})^2 - 2(\overline{BC})\ (\overline{BF})$$

$$= (\overline{AB})^2 + (\overline{CD})^2 + 2(\overline{BC})^2 + 2(\overline{BC})\ \ (\overline{CE})^2 - 2(\overline{BC})\ (\overline{BF})$$

In parallelogram opposite sides are congruent, so

$$\overline{AB} = \overline{DC},\ \ \overline{AD} = \overline{BC},\ \text{ and } \ \overline{BF} = \overline{CE}$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + (\overline{AB})^2\ 2(\overline{BC})^2 + 2\ (\overline{CE}) - 2\ (\overline{BC})\ \overline{CE}$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + 2(\overline{BC})^2$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$$

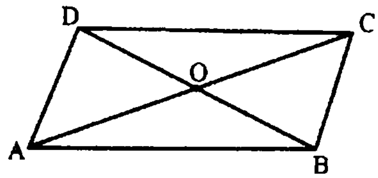
Hence Proved.

Q4. Prove that the sum of the squares of the sides of a parallelogram is equal to sum of the squares of its diagonals.

Solution:

Given:

ABCD is a parallelogram with $\overline{AC}$ and $\overline{BD}$ are its diagonals.



To Prove

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2 = \overline{AC}^2 + \overline{BD}^2$$

In ΔACD

$$\overline{DC}^2 + \overline{AD}^2 = 2\overline{OD}^2 + \overline{OA}^2 \qquad \text{\_\_\_\_\_\_ (i)}$$

And In ΔABC

$$\overline{AB}^2 + \overline{BC}^2 = 2\overline{OB}^2 + \overline{OA}^2 \qquad \text{\_\_\_\_\_\_ (ii)}$$

Adding (i) & (ii)

$$\overline{DC}^2 + \overline{AD}^2 + \overline{AB}^2 + \overline{BC}^2 = 4\overline{OA}^2 + 2\overline{OD}^2 + 2\overline{OB}^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = 4\overline{OA}^2 + 2\overline{OD}^2 + 2\overline{OD}^2 \qquad \left[\because \overline{OB} = \overline{OD}\right]$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = 4\overline{OA}^2 + 4\overline{OD}^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = \left(2\overline{OA}\right)^2 + \left(2\overline{OD}\right)^2$$

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = \overline{AC}^2 + \overline{BD}^2$$

Hence proved

SOLVED MISCELLANEOUS EXERCISE 8

Q1. In a ΔABC, $m\angle A = 60°$, prove that $(\overline{BC})^2 = (\overline{AB})^2 + \overline{AC}^2 - m\,\overline{AB}\,.\,m\,\overline{AC}$.

Solution:

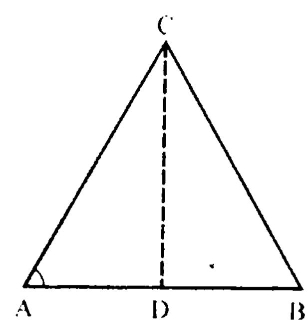
In a ΔABC, $m\angle A = 60°$,

Given:

In a ΔABC, $m\angle A = 60°$

Required:

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \overline{AB}.\overline{AC}$$



Construction:

Draw $\overline{CD} \perp \overline{AB}$, so that the Projection of $\overline{AC}$ on AB.

Proof:

In right angle ΔACD

$\angle A = 60^\circ$ and $\angle ACD = 30^\circ$ (being complement of $\overline{CA}$)

And $\angle$ ACD, side opposite to $\angle = 30^\circ = \frac{1}{2}$ hyp $\overline{AC}$.

Now, according to the theorem, we have

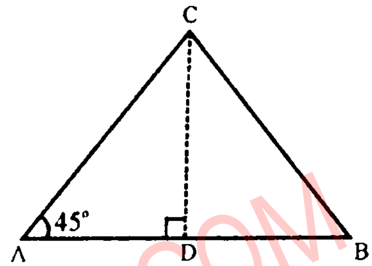
$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\,\overline{AB}\,.\,\overline{AD}$

$\Rightarrow \overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \overline{AB}\,.\,\overline{AC}$ $[\because 2AD = AC]$.

Q2. In a ΔABC, $m\angle A = 45^\circ$, prove that $(\overline{BC})^2 = (\overline{AB})^2 + (\overline{AC})^2 - \sqrt{2}\ m\overline{AB}.m\overline{AC}$.

Solution:

In a ΔABC, m $\angle A = 45^\circ$.



Given:

In a ΔABC; $m\angle A = 45^\circ$.

Required:

$\overline{BC}^2 = AB^2 + AC^2 - \sqrt{2}\,\overline{AB}.\overline{AC}^2$

Construction:

Draw CD $\perp$ AB, so that the projection of AC on AB.

Proof:

In right angle ΔACD

$\angle A = 45^\circ$ and $\angle ACD = 45^\circ$ (being complement of $\angle A$)

And $\angle ACD$; side opposite to $\angle 45^\circ = \sqrt{2}$ .hyp. $\overline{AC}$

ΔABC is acute angled at A, so according to the theorem, we have

$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}.\overline{AD}$

$\Rightarrow BC^2 = AB^2 + AC^2 - \sqrt{2}\ \overline{AB}.\overline{AD}$ $\left[\because\ 2\overline{AD} = \sqrt{2}AC\right]$

Hence proved

Q3. In a ΔABC, calculate mBC when m$\overline{AB}$ = 5 cm, m$\overline{AC}$ = 4 cm, $m\angle A = 60^\circ$.

Solution:

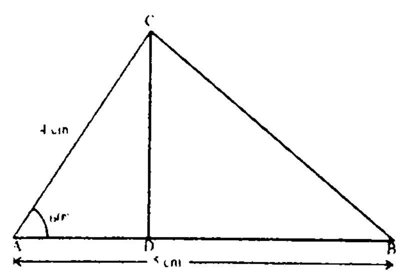
We know that

$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \overline{AB}^2.AC$

$= 5^2 + 4^2 - 5.4$

$= 25 + 16 - 20$

$= 21$



$m\overline{BC}^2 = \sqrt{21} = 4.58\text{cm}$

Q4. In a ΔABC, calculate m $\overline{AC}$ when m $\overline{AB}$ = 5 cm, mBC = $4\sqrt{2}$ cm, m∠B = 45°.

Solution:

We know that

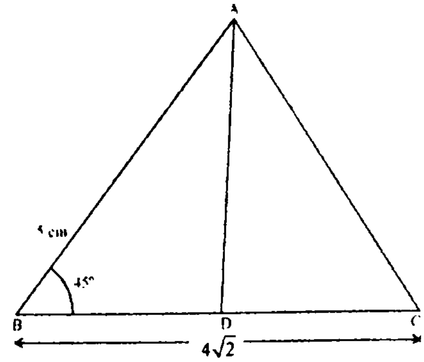
$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - \sqrt{2}\,\overline{AB}.\overline{BC}$$

$$= (5)^2 + \left(4\sqrt{2}\right)^2 - \sqrt{2}(5)\left(4\sqrt{2}\right)$$

$$= 25 + 32 - 40$$

$$= 57 - 40 = 17.$$

$$m\overline{AC}^2 = \sqrt{17}\text{cm} = 4.123\text{cm}$$



Q5. In a triangle ABC, m $\overline{BC}$ = 21 cm, m $\overline{AC}$ = 17 cm, m $\overline{AB}$ = 10 cm. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

C = 10 cm, a = 21 cm, b = 17 m, x = ?

We know that

$$C^2 = a^2 + b^2 - 2(a)(x)$$

$$(10)^2 = (21)^2 + (17)^2 - 2(21)(x)$$

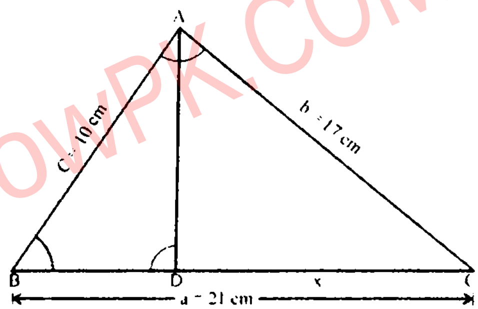
$$100 = 441 + 189 - 42x$$

$$42x = 441 + 189 - 42x$$

$$42x = 730 - 100$$

$$42x = 630$$

$$x = \frac{630}{40} = 15 \text{ cm}$$



Q6. In a triangle ABC, mBC = 21 cm. mAC = 17 cm, mAB = 10 cm. Calculate the projection of AB upon BC.

Solution:

C = 10 cm, a = 21 cm, b = 17 m, x = ?

We know that

$$b^2 = a^2 + c^2 - 2ax$$

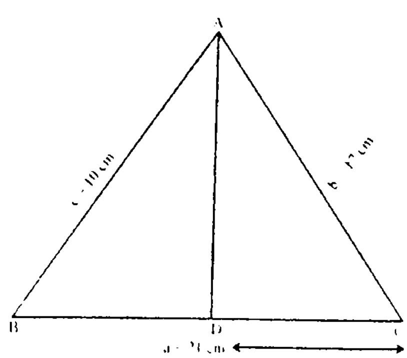
$$(17)^2 = (10)^2 + (21)^2 - 2(21)(x)$$

$$289 = 100 + 441 - 42x$$

$$289 = 541 - 42x$$

$$42x = 541 - 289$$

$$42x = 252$$



$$x = \frac{252}{42} = 6 \text{ cm}$$

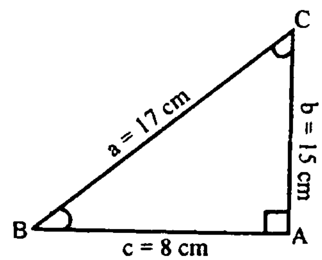
Q7. In a ΔABC, a = 17 cm, b =15 cm and c = 8 cm find $m\angle A$.

Solution:

Given:

In a ΔABC, a = 17 cm, b = 15 cm and, c = 8 cm

Required: $m\angle A = ?$



by Pythagoras theorem.

$$a^2 = b^2 + c^2$$

$$17^2 = 15^2 + 8^2$$

$$289 = 225 + 64$$

$$289 = 289$$

So, it satisfied, that given values are the sides of a right angled triangle.

$\therefore m\angle A = 90^\circ$

Q8. In a ΔABC, a = 17 cm, b = 15 cm and c = 8 cm find $m\angle B$.

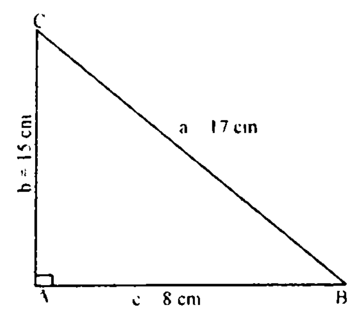
Solution:

Given:

In a Δ ABC; a = 17 cm, b = 15 cm and c = 8 cm

Required:

$m\angle B = ?$



We know that it is right angled triangle.

$$\text{Sin}(m\angle B) = \frac{b}{a} = \frac{15}{17} = 0.882$$

$$m\angle B = \sin^{-1}(0.882) = 61.90$$

Q9. Whether the triangle with sides 5 cm, 7 cm, 8 cm is acute, obtuse or right angled.

Solution:

Given:

a = 5cm; b= 7 cm; c = 8 cm

Case I:

$$c^2 = a^2 + b^2$$

$$8^2 = 5^2 + 7^2$$

$$64 = 25 + 49$$

$$64 = 74$$

It is not right angled triangle.

Case II:

Then.

$$b^2 = a^2 + c^2$$
$$(7)^2 = (5)^2 + (8)^2$$
$$49 = (5)^2 + (8)^2$$
$$49 \neq 91$$

Case III:

$$a^2 = b^2 + c^2$$
$$(5)^2 = (7)^2 + (8)^2$$
$$25 = 49 + 64$$
$$25 \neq 113$$

Which is not possible, so the given data shows that it is not obtuse triangle; It is acute angled triangle.

Q10. Whether the triangle with sides 8 cm, 15 cm, 17 cm is acute, obtuce or right angled.

Solution:

$a = 8;\quad b = 15;\quad c = 17$

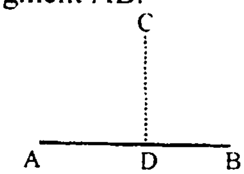
Case1: It is right angled.

$$c^2 = a^2 + b^2$$
$$17^2 = 8^2 + 15^2$$
$$289 = 64 + 225$$
$$289 = 289$$

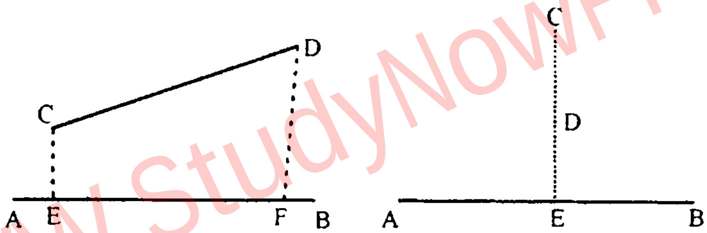
Hence, it is right angled triangle.

SUMMARY

- The projection of a given point on a line segment is the foot ⊥ of drawn from the point on that line segment. If $\overline{CD} \perp \overline{AB}$, then evidently D is the foot of perpendicular $\overline{CD}$ from tire point C on the line segment AB.



- The projection of a line segment $\overline{CD}$ on a line segment AB is the portion $\overline{EF}$ of the latter intercepted between foots of the perpendiculars drawn from C and D. However projection of a vertical line segment $\overline{CD}$ on a line segment AB is a point on $\overline{AB}$ which is of zero dimension.



- In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.
- In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.
- In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apolloniu's Theorem).

✪✪✪✪✪✪✪✪✪✪✪✪