QUADRATIC EQUATION

In this unit, students will learn how to:

- define quadratic equation.
- solve a quadratic equation in one variable by factorization.
- solve a quadratic equation in one variable by completing square.
- derive quadratic formula by using method of completing square.
- solve a quadratic equation by using quadratic formula.
- solve the equations of the type $ax^4 + bx^2 + c = 0$ by reducing it to the quadratic form.
- solve the equations of the type $a\,p(x) + \dfrac{b}{p(x)} = c$.
- solve reciprocal equations of the type $a\left(x^2 + \dfrac{1}{x^2}\right) + b\left(x + \dfrac{1}{x}\right) + c = 0$
- solve exponential equations involving variables in exponents.
- solve equations of the type $(x + a)(x + b)(x + c)(x + d) = k$ where $a + b = c + d$.
- solve radical equations of the types
  - (i) $\sqrt{ax + b} = cx + d$,
  - (ii) $\sqrt{x + b} + \sqrt{x + b} = \sqrt{x + c}$,
  - (iii) $\sqrt{x^2 + px + m} + \sqrt{x^2 + px + n} = q$

Quadratic Equation:

An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.

A second degree equation in one variable x of the form

$ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, c are real numbers, is called the general or standard form of a quadratic equation.

Here a is the co-efficient ofx^2, b is the co-efficient of x and constant term is c.

The equations $x^2 - 7x + 6 = 0$ and $3x^2 - 4x = 5$ are the examples of quadratic equations $x^2 - 7x + 6 = 0$ is m standard form but $3x^2 + 4x = 5$ is not in standard form.

If $b = 0$ in a quadratic equation $ax^2 + bx + c = 0$, then it is called a pure quadratic equation. For example $x^2 - 16 = 0$ and $4x^2 = 7$ are the pure quadratic equations.

Solution of quadratic equations:

To find solution set of a quadratic equation, following methods are used:

(i) factorization (ii)completing square

SOLVED EXERCISE 1.1

1. Write the following quadratic equations in the standard form and point out pure quadratic equations.

(i) $(x + 7)(x - 3) = -7$

Solution:

$$(x + 7)(x - 3) = -7$$
$$x(x - 3) + 7(x - 3) = -7$$
$$x^2 - 3x + 7x - 21 = -7$$
$$x^2 + 4x - 21 + 7 = 0$$
$$x^2 + 4x - 14 = 0$$

The above equation is a quadratic equation.

(ii) $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

Solution:

$$\frac{x^2 + 4}{3} - \frac{x}{7} = 1$$

Multiply both sides by 21, we get

$$21 \times \frac{x^2 + 4}{3} - 21 \times \frac{x}{7} = 1 \times 21$$
$$7(x^2 + 4) - 3x = 21$$
$$7x^2 + 28 - 3x = 21$$
$$7x^2 - 3x + 28 - 21 = 0$$
$$7x^2 - 3x + 7 = 0$$

(iii) $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Solution:

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$
$$\frac{x^2 + (x^2 + 1)^2}{x(x+1)} = 6$$
$$x^2 + x^2 + 2x + 1 = 6x(x + 1)$$
$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$2x^2-6x^2+2x-6x+1=0$$
$$-4x^2-4x+1=0$$
$$-(4x^2+4x-1)=0$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The above equation is a quadratic equation.

(iv) $\dfrac{x+4}{x-2}-\dfrac{x-2}{x}+4=0$

Solution:

$$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$$
$$\frac{x(x+4)-(x-2)^2+4x(x-2)}{x(x-2)}$$
$$\Rightarrow \quad (x^2+4x)-(x^2-4x+4)+4(x^2-8x)=0$$
$$x^2+4x-x^2+4x-4+4x^2-8x=0$$
$$x^2-x^2+4x^2+4x+4x-8x-4=0$$
$$4x^2+8x-8x-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0 \quad \Rightarrow \quad x^2-1=0$$

The above equation is a pure quadratic equation.

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution:

$$\frac{x+3}{x+4}-\frac{x-5}{x}=1$$
$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$

$$\left(x^2+3x\right)-x(x-5)-4(x-5)=x(x+4)$$
$$x^2+3x-x^2+5x-4x+20=x^2+4x$$
$$x^2-x^2+3x+5x-4x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$-x^2+4x-4x+20=0$$
$$-x^2-20=0$$
$$-\left(x^2-20\right)=0 \Rightarrow x^2-20=0$$

The above equation is a pure quadratic equation.

(vi) $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$
$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{x(x+3)+1(x+3)+\left(x^2+4x+4\right)}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$$
$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$
$$25\left(x^2+5x+6\right)=12\left(2x^2+8x+7\right)$$
$$25x^2+125x+150=24x^2+96x+84$$
$$x^2+29x+66=0$$

The above equation is a pure quadratic equation.

2. Solve by factorization:

(i) $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$
$$x^2-5x+4x-20=0$$
$$x(x-5)+4(x-5)=0$$
$$(x+4)(x-5)=0$$

(ii) $3y^2 = y(y-5)$

Solution:

$$3y^2 = y(y-5)$$
$$3y^2 = y^2 - 5y$$
$$3y^2 - y^2 + 5y = 0$$
$$2y^2 + 5y = 0$$
$$y(2y+5) = 0$$

Either $y = 0$ or $2y + 5 = 0$

$$2y = -5$$
$$y = -\frac{5}{2}$$

Thus, solution set $= \left\{0, -\frac{5}{2}\right\}$

(iii) $4 - 32x = 17x^2$

Solution:

$$4 - 32x = 17x^2$$

or
$$17x^2 + 32x - 4 = 0$$
$$17x^2 + 34x - 2x - 4 = 0$$
$$17x(x+2) - 2(x+2) = 0$$
$$(17x-2)(x+2) = 0$$

Either $17x - 2 = 0$ or $x + 2 = 0$

$17x = 2$ $\quad x = -2$

$$x = \frac{2}{17}$$

Thus, solution set $= \left\{\frac{2}{17}, -2\right\}$

(iv) $x^2 - 11x = 152$

Solution:

$$x^2 - 11x = 152$$
$$x^2 - 11x - 152 = 0$$
$$x^2 - 19x + 8x - 152 = 0$$
$$x(x-19) + 8(x-19) = 0$$
$$(x+8)(x-19) = 0$$

Either $x + 8 = 0$ or $x - 19 = 0$

$x = -8$ $\quad x = 19$

Thus, solution set $= \{-8, 19\}$

(v) $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

Solution:

$$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

$$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$$

$$\frac{x^2+2x+1+x^2}{x^2+x}=\frac{25}{12}$$

$$\frac{2x^2+2x+1}{x^2+x}=\frac{25}{12}$$

$$25(x^2+x)=12(2x^2+2x+1)$$

$$25x^2+25x=24x^2+24x+12$$

$$25x^2-24x^2+25x-24x-12=0$$

$$x^2+x-12=0$$

$$x^2+4x-3x-12=0$$

$$x(x+4)-3(x+4)=0$$

$$(x-3)(x+4)=0$$

Either $x-3=0$ or $x+4=0$

$x=3$ $\quad$ $x=-4$

Thus, solution set = $\{3,-4\}$

(iv) $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

Solution:

$$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$

$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-7x+12}$$

$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$

$$2(x^2-x+12)=-1(x-9)$$

$$2x^2-14x+24=-x+9$$

$$2x^2-14x+x+24-9=0$$

$$2x(x-5)-3(x-5)=0$$

$$(2x-3)(x-5)=0$$

Either $2x-3=0$ or $x-5=0$

$2x=3$ $\quad$ $x=5$

$x=\frac{3}{2}$

Thus, Solution set $=\left\{5,\frac{3}{2}\right\}$

Q3. Solve the following equations by completing square:

(i) $7x^2+2x-1=0$

Solution:

$$7x^2+2x-1=0$$

$$7x^2+2x=1$$

$$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$$

$$x^2+\frac{2x}{7}=\frac{1}{7}$$

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root on both sides, we get

$$x+\frac{1}{7}=\pm\sqrt{\frac{8}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1\pm2\sqrt{2}}{7}$$

Thus, solution set $=\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

(ii) $ax^2+4x-a=0$

Solution:

$$ax^2+4x-a=0$$

$$ax^2 + 4x = a$$

$$\frac{ax^2}{a} + \frac{4x}{a} = \frac{a}{a}$$

$$x^2 + \frac{4x}{a} = 1$$

$$(x)^2 + 2(x)\left(\frac{2}{a}\right) + \left(\frac{2}{a}\right)^2 = 1 + \left(\frac{2}{a}\right)^2$$

$$\left(x + \frac{2}{a}\right)^2 = 1 + \frac{4}{a^2}$$

$$\left(x + \frac{2}{a}\right)^2 = \frac{a^2 + 4}{a^2}$$

Taking square root on both sides, we get

$$x + \frac{2}{a} = \pm\sqrt{\frac{a^2 + 4}{a^2}}$$

$$x = -\frac{2}{a} \pm \frac{\sqrt{a^2 + 4}}{a}$$

$$x = \frac{-2 \pm \sqrt{a^2 + 4}}{a}$$

Thus, solution set $= \left\{\frac{-2 \pm \sqrt{a^2 + 4}}{a}\right\}$

(iii) $11x^2 - 34x + 3 = 0$

Solution:

$$11x^2 - 34x + 3 = 0$$

$$11x^2 - 34x = -3$$

$$\frac{11x^2}{11} - \frac{34}{11}x = -\frac{3}{11}$$

$$x^2 - \frac{34}{11}x = -\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$$

$$\left(x-\frac{34}{22}\right)^2=-\frac{3}{11}+\frac{1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{132+1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$$

Taking square root on both sides we get

$$\left(x-\frac{34}{22}\right)^2=\pm\sqrt{\frac{1024}{484}}$$

$$x-\frac{34}{22}=\pm\frac{32}{22}$$

$$x=\frac{34}{22}\pm\frac{32}{22}$$

$$x=\frac{34\pm 32}{22}$$

$$x=\frac{34+32}{22},\ x=\frac{34-32}{22}$$

$$=\frac{66}{22} \qquad =\frac{2}{22}$$

$$=3 \qquad =\frac{1}{11}$$

Thus, solution set $\left\{3,\frac{1}{11}\right\}$

(iv) $\ell x^2 - mx + n = 0$

Solution:

$$\ell x^2 - mx + n = 0$$

$$\ell x^2 + mx = -n$$

$$\frac{\ell x^2}{\ell}+\frac{mx}{\ell}=-\frac{n}{\ell}$$

$$x^2+\frac{mx}{\ell}=-\frac{n}{\ell}$$

$$(x)^2+2(x)\left(\frac{m}{2\ell}\right)+\left(\frac{m}{2\ell}\right)^2=-\frac{n}{\ell}+\left(\frac{m}{2\ell}\right)^2$$

$$\left(x+\frac{m}{2\ell}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2=\frac{-4\ell n+m^2}{4\ell^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2=\frac{m^2-4\ell n}{4\ell^2}$$

Taking square root an both sides, we get

$$\sqrt{\left(x+\frac{m}{2\ell}\right)^2}=\pm\frac{\sqrt{m^2-4\ell n}}{4\ell^2}$$

$$x+\frac{m}{2\ell}=\pm\frac{\sqrt{m^2-4ln}}{2\ell}$$

$$x=\frac{m}{2\ell}\pm\frac{\sqrt{m^2-4\ell n}}{2\ell}$$

$$x=\frac{-m\pm\sqrt{m^2-4\ell n}}{2\ell}$$

Thus, solution set $=\left\{\frac{-m\pm\sqrt{m^2-4\ell n}}{2\ell}\right\}$

(v) $3x^2+7x=0$

Solution:

$$3x^2+7x=0$$

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\frac{7}{3}x=0$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking square root on both sides, we get.

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\left(\frac{7}{6}\right)^2}$$

$$x+\frac{7}{6}=\pm\frac{7}{6}$$

$$x=-\frac{7}{6}\pm\frac{7}{6}$$

$$x=-\frac{7}{6}+\frac{7}{6} \quad \text{or} \quad x=-\frac{7}{6}-\frac{7}{6}$$

$$x=0 \qquad\qquad x=-\frac{14}{6}$$

$$x=-\frac{7}{3}$$

Thus, solution set = $\left\{0,-\frac{7}{3}\right\}$

(vi) $x^2-2x-195=0$

Solution:

$$x^2-2x-195=0$$

$$x^2-2x=195$$

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root on both sides, we get

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm 14$$

$$x=1\pm 14$$

$$x=1\pm 14 \quad \text{or} \quad x=1-14$$

$$=15 \qquad\qquad =-13$$

Thus, solution set = $\{-13,15\}$

(vii) $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution:

$$-x^2+\frac{15}{2}=\frac{7}{2}x$$

$$-x^2-\frac{7}{2}x=-\frac{15}{2}$$

$$-\left(x^2+\frac{7}{2}x\right)=-\frac{15}{2}$$

$$\Rightarrow \quad x^2+\frac{7}{2}x=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=-\frac{7}{4}\pm\frac{13}{4}$$

$$x=-\frac{7}{4}+\frac{13}{4} \quad \text{or} \quad x=-\frac{7}{4}-\frac{13}{4}$$

$$x=\frac{6}{4} \qquad x=-\frac{20}{4}$$

$$x=\frac{3}{2} \qquad x-5$$

(viii) $x^2+17x+\frac{33}{4}=0$

Solution:

$$x^2+17x+\frac{33}{4}=0$$

$$x^2+17x=-\frac{33}{4}$$

$$(x)^2+2(x)\left(\frac{17}{2}\right)+\left(\frac{17}{2}\right)^2=-\frac{33}{4}+\left(\frac{17}{2}\right)^2$$

$$\left(x+\frac{17}{2}\right)^2=-\frac{33}{4}+\frac{289}{4}$$

$$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$$

Taking square root on both sides,

$$\sqrt{\left(x+\frac{17}{2}\right)^2}=\pm\sqrt{\frac{256}{4}}$$

$$x+\frac{17}{2}=\pm\frac{16}{2}$$

$$x=-\frac{17}{2}\pm\frac{16}{2}$$

$$x=-\frac{17}{2}+\frac{16}{2} \quad \text{or} \quad x=-\frac{17}{2}-\frac{16}{2}$$

Thus, solution set $=\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

(ix) $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$

Solution:

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$

$$\frac{4(3x+1)-8}{3x+1}=\frac{3x^2+5}{3x+1}$$

$$\frac{12x+4-8}{3x+1}=\frac{3x^2+5}{3x+1}$$

$$\frac{12x-4}{3x+1}=\frac{3x^2+5}{3x+1}$$

Multiplying both sides by (3x + 1), we get

$$12x-4=3x^2+5$$

or $$3x^2+5-12x+4=0$$

$$3x^2-12x+9=0$$

$$3(x^2-4x+3)=0$$

$\Rightarrow$ $$x^2-4x+3=0$$

$$x^2-3x-x+3=0$$

$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ or $x-3=0$

$x=1$ $\quad$ $x=3$

Thus, solution set = $\{1, 3\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution:

$$7(x+2a)^2+3a^2 = 5a(7x+23a)$$
$$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$$
$$7x^2+28ax+28a^2+3a^2=35ax+115a^2$$
$$7x^2-7ax-84a^2=0$$
$$7(x^2-ax-12a^2)=0$$
$$x^2-ax-12a^2=0$$
$$x^2-ax=12a^2$$

$$\Rightarrow \quad (x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a}{2}+\frac{7a}{2}, x=\frac{a}{2}-\frac{7a}{2}$$
$$=\frac{8a}{2}=-\frac{6a}{2}$$
$$=4a=-3a$$

Thus, solution set = $\{-3a, 4a\}$

Quadratic Formula:

Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$ax^2 + bx + c = 0, \quad a \neq 0$

Dividing each term of the equation by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding $\left(\frac{b}{a}\right)^2$ on both sides, we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

or $$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square root of both sides, we get

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

or $$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is known as "quadratic formula".

SOLVED EXERCISE 1.2

Q1. Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution:

$$2 - x^2 = 7x$$
$$-x^2 - 7x + 2 = 0$$
$$-(x^2 + 7x - 2) = 0$$
$$\Rightarrow \quad x^2 + 7x - 2 = 0$$

Compare it with, we have

$ax^2 + bx + c = 0$

Here $a = 1, b = 7, c = -2$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

Thus, solution set $= \left\{\dfrac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $8x + 1 = 0$

Solution:

$$5x^2 + 8x + 1 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here $a = 5, b = 8, c = 1$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{2(-4 \pm \sqrt{11})}{10}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Thus, solution set = $= \left\{\dfrac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}\,x^2 + x = 4\sqrt{3}$

Solution:

$$\sqrt{3}\,x^2 + x = 4\sqrt{3}$$

$$\sqrt{3}\,x^2 + x - 4\sqrt{3} = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here $a = \sqrt{3}, b = 1, c = -4\sqrt{3}$

Now $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1-48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$x = \dfrac{-1 \pm 7}{2\sqrt{3}}$ or $x = \dfrac{-1-7}{2\sqrt{3}}$

$x = \dfrac{6}{2\sqrt{3}}$ $\quad$ $x = \dfrac{-8}{2\sqrt{3}}$

$x = \dfrac{3}{\sqrt{3}}$ $\quad$ $x = -\dfrac{4}{\sqrt{3}}$

$$x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{3\sqrt{3}}{(\sqrt{3})^2}$$

$$x = \frac{3\sqrt{3}}{3}$$

$$x = \sqrt{3}$$

Thus, solution set = $\left\{\sqrt{3}, -\dfrac{4}{\sqrt{3}}\right\}$

(iv) $4x^2 - 14 = 3x$

Solution:

$$4x^2 - 14 = 3x$$

$$4x^2 - 3x - 14 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + c = 0$$

Here $a = 4, b = -3, c = -14$

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

Thus, solution set $= \left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $6x^2 - 3 - 1x = 0$

Solution:

$$6x^2 - 3 - 1x = 0$$

$$6x^2 - 7x - 3 = 0$$

Compare it with, we have

$$ax^2 + bx + c = 0$$

Here $a = 6, b = -7, c = -3$

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$x=\frac{7+11}{12} \quad , \quad x=\frac{7-11}{12}$$

$$=\frac{18}{12} \quad\quad =-\frac{4}{12}$$

$$=\frac{3}{2} \quad\quad =-\frac{1}{3}$$

Thus, solution set = $\left\{-\frac{1}{3},\frac{3}{2}\right\}$

(vi) $3x^2+8x+2=0$

Solution:

$$3x^2+8x+2=0$$

Compare it with

$$ax^2+bx+c=0$$

Here $a=3, b=8, c=2$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$$

$$x=\frac{-8\pm\sqrt{64-24}}{6}$$

$$x=\frac{-8\pm\sqrt{40}}{6}$$

$$x=\frac{-8\pm\sqrt{10}}{6}$$

$$x=\frac{2\left(-4\pm\sqrt{10}\right)}{6}$$

$$x=\frac{-4\pm\sqrt{10}}{3}$$

Thus, solution set = $\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution:

$$\frac{3}{x-6}-\frac{4}{x-5}=1$$

$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3x-15-4x+24=(x-6)(x-5)$$

$$-x+9=x^2-11x+30$$

$$x^2-11x+x+30-9=0$$

$$x^2-10x+21=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=1, b=-10, c=21$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{-8\pm4}{2}$$

$$x=\frac{10+4}{2},\quad x=\frac{10-4}{2}$$

$$x=\frac{14}{2}\quad\quad x=\frac{6}{2}$$

$$x=7\quad\quad x=3$$

Thus, solution set = {3, 7}

(viii) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

Solution:

$$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$$

$$\frac{2x(x+2)-(x-1)(4-x)}{2x(x-1)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-x^2-4+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x+x^2-5x+4}{2x^2-2x}=\frac{7}{3}$$

$$7(2x^2-2x)=3(3x^2-x+4)$$

$$14x^2-14x=9x^2-3x+12$$

$$14x^2-9x^2-14x+3x-12=0$$

$$5x^2-11x-12=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=5, b=-11, c=-12$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121-240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$x=\frac{11+19}{10}$, $\quad x=\frac{11-10}{10}$

$x=\frac{30}{10}$ $\quad x=\frac{-8}{10}$

$x=3$ $\quad x=-\frac{4}{5}$

Thus, solution set = $\left\{3,-\frac{4}{5}\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$

Solution:

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$

$$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)}=2$$

$$ax-a^2+bx-b^2=2(x-a)(x-b)$$

$$ax+bx-a^2-b^2=2(x^2-ax-bx+ab)$$

$$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-3(a+b)x+(2ab+a^2+b^2)=0$$

$$2x^2-3(a+b)x+(a+b)^2=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=2,\ b=-3\ (a+b)\ ;\ c=(a+b)^2$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$$x=\frac{3(a+b)+(a+b)}{4},\qquad x=\frac{3(a+b)-(a+b)}{4}$$

$$x=\frac{3a+3b+a+b}{4}\qquad x=\frac{3a+3b-a-b}{4}$$

$$x=\frac{4a+4b}{4}\qquad x=\frac{2a+2b}{4}$$

$$x=\frac{4(a+b)}{4}\qquad x=\frac{2(a+b)}{4}$$

$$x=a+b,\qquad x=\frac{1}{2}(a+b)$$

Thus, solution set $=\left\{(a+b),\frac{1}{2}(a+b)\right\}$

(x) $-(\ell+m)-\ell x^2+(2\ell+m)x=0$

Solution:

$$-(\ell+m)-\ell x^2+(2\ell+m)x=0, \ell=0$$

$$-\ell x^2+(2\ell+m)x-(\ell+m)=0$$

$$-[\ell x^2-(2\ell+m)x+(\ell+m)]=0$$

$$\Rightarrow \ell x^2-(2\ell+m)x+(\ell+m)=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=\ell$, $b=-(2\ell+m)$, $c=(\ell+m)$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-\left[-(2\ell+m)\pm\sqrt{[-(2\ell+m)]^2-4(\ell)(\ell+m)}\right]}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{4\ell^2+4\ell m+m^2-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$$x=\frac{2\ell+2m}{2\ell}, \quad x=\frac{2\ell+2m-m}{2\ell}$$

$$x=\frac{2\ell+2m}{2l} \qquad =\frac{2l}{2l}$$

$$=\frac{2(l+m)}{2\ell} \qquad =\ell$$

$$=\frac{\ell+m}{\ell}$$

Thus, solution set $=\left\{\ell, \dfrac{\ell+m}{\ell}\right\}$

SOLVED EXERCISE 1.3

Q1. Solve the following equations.

(1) $2x^4-11x^2-5=0$

Solution:

$2x^4 - 11x^2 - 5 = 0$ ——— (i)

Let $x^2 = y$. then $x^4 = y^2$

So eq. (i) becomes

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(2y-1)(y-5) = 0$

Either $2y - 1 = 0$ or $y - 5 = 0$

$2y = 1$ $y = 5$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

$x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 5$ in $x^2 = y$, we get

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ——— (i)

Let $x^2 = y$. then $x^4 = y^2$

So eq. (i) becomes

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(2y-1)(y-4) = 0$

Either $2y - 1 = 0$ or $y - 4 = 0$

$2y = 1$ $y = 4$

$y = \frac{1}{2}$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

Put $y = 4$ in $x^2 = y$, we get

$$x^2 = y$$
$$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$$
$$x = \pm\frac{1}{\sqrt{2}}$$

$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(3) $5x^{1/2} = 7x^{1/4} - 2$

Solution:

$$5x^{1/2} = 7x^{1/4} - 2$$
$$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \text{\_\_\_\_\_\_(i)}$$

Let $x^{\frac{1}{4}} = y.\text{then } x^{\frac{1}{2}} = y^2$

So eq. (1) becomes

$$5y^2 - 7y + 2 = 0$$
$$5y^2 - 5y - 2y + 2 = 0$$
$$5y(y-1) - 2(y-1) = 0$$
$$(5y-2)(y-1) = 0$$

Either $5y - 2 = 0$ or $y - 1 = 0$

$$5y = 2 \qquad y = 1$$
$$y = \frac{2}{5}$$

Put $y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$, we get

$$x^{\frac{1}{4}} = y$$
$$x^{\frac{1}{4}} = \frac{2}{5}$$

Taking power '4' on both sides, we get

$$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{3}\right)^4$$
$$x = \frac{2^4}{5^4}$$
$$x = \frac{16}{625}$$

Put $y = 1$ in $x^{\frac{1}{2}} = y$, we get

$$x^{\frac{1}{4}} = y$$
$$x^{\frac{1}{4}} = 1$$

Taking power '4' on both sides, we get

$$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$$
$$\left(x^{\frac{1}{4}}\right)^4 = 1$$
$$x = 1$$

Thus, solution set $= \left\{\frac{16}{625}, 1\right\}$

(1) $\mathbf{x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}}$

Solution:

$$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$

$$x^{\frac{2}{3}} - 15x^{\frac{1}{3}} + 54 = 0 \_\_\_\_\_\_ (i)$$

Let $x^{\frac{1}{2}} = y$. Then $x^{\frac{2}{3}} = y^2$

So eq (i) becomes

$$y^2 - 15y + 54 = 0$$

$$y^2 - 9y - 6y + 54 = 0$$

$$y(y-9) - 6(y-9) = 0$$

$$(y-6)(y-9) = 0$$

Either $y - 9 = 0$ or $y - 6 = 0$

$y = 9$ $\quad$ $y = 6$

Put $y = 9$ in $x^{\frac{1}{3}} = y$, we get

$$x^{\frac{1}{3}} = y$$

$$x^{\frac{1}{3}} = 9$$

Taking cube on both

We get

$$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$$

$$x = 729$$

Put $y = 6$ in $x^{\frac{1}{3}} = y$, we get

$$x^{\frac{1}{3}} = y$$

$$x^{\frac{1}{3}} = 6$$

Taking cube on both

We get

$$\left(x^{\frac{1}{3}}\right)^3 = (6)^3$$

$$x = 216$$

Thus, solution set = {729, 216}

(5) $\mathbf{3x^{-2} + 5 = 8x^{-1}}$

Solution:

$$3x^{-2} + 5 = 8x^{-1}$$

$$3x^{-2} - 8x^{-1} + 5 = 0 \_\_\_\_\_\_ (i)$$

Let $x^{-1} = y =$ Then $x^{-2} = y^2$

So eq. (i) becomes

$$3y^2 - 8y + 5 = 0$$

$$3y^2 - 5y - 3y + 5 = 0$$

$$y(3y - 5) - 1(3y - 5) = 0$$

$$(y - 1)(3y - 5) = 0$$

Either $y - 1 = 0$ or $3y - 5 = 0$

$y = 1$ $\qquad$ $3y = 5$

$$y = \frac{5}{3}$$

Put $y = 1$ in $x^{-1} = y$, we get

$$x^{-1} = y$$
$$x^{-1} = 1$$
$$\frac{1}{x} = 1$$
$$\text{or } x = 1$$

Put $y = \frac{5}{3}$ in $x^{-1} = y$, we get

$$x^{-1} = y$$
$$x^{-1} = \frac{5}{3}$$
$$\frac{1}{x} = \frac{5}{3}$$
$$\text{or } x = \frac{5}{3}$$

Thus, solution set $= \left\{1, \frac{3}{5}\right\}$

6. $(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4$

Solution:

$$(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4$$

$$(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4 \quad \text{\_\_\_\_(i)}$$

Let $2x^2 + 1 = y$

So eq. (i) becomes

$$y + \frac{3}{y} = 4$$

Multiplying both sides by 'y', we get

$$y^2 + 3 = 4y$$
$$y^2 - 4y + 3 = 0$$
$$y^2 - 3y - y + 3 = 0$$
$$y(y - 3) - 1(y - 3) = 0$$
$$(y - 1)(y - 3) = 0$$

Either $y - 1 = 0$ or $y - 3 = 0$

$y = 1$ $\qquad$ $y = 3$

Put $y = 1$ in $2x^2 + 1 = y$, we get $\qquad$ Put $y = 3$ in $2x^2 + 1 = y$, we get

$$2x^2+1=1$$
$$2x^2+1=1-1$$
$$2x^2=0$$
$$x^2=0$$
$$\Rightarrow \quad x=0$$

$$2x^2+1=3$$
$$2x^2+1=3-1$$
$$2x^2=2$$
$$x^2=1$$
$$\Rightarrow \quad x=\pm 1$$

Thus, solution set = $\{-1, 0, 1\}$

(7) $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

Solution:

$$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \quad \text{———— (i)}$$

Let $\frac{x}{x-3}=y$

So eq. (i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by 'y', we get

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$(y)^2-2(y)(2)+(2)^2=0$$
$$(y-2)^2=0$$
$$\Rightarrow y-2=0$$

Put $y=2$ in $\frac{x}{x-3}=y$, we get

$$\frac{x}{x-3}=y$$
$$\frac{x}{x-3}=2$$
$$2(x-3)=x$$
$$2x-6=x$$
$$2x-x=6$$
$$x=6$$

Thus, solution set = $\{6\}$

8. $\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=2\frac{1}{6}$

Solution:

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=2\frac{1}{6}$$

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=\frac{13}{6} \quad \text{———}(i)$$

Let $\frac{4x+1}{4x-1}=y$

So eq. (i) becomes

$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by '6y', we get

$$6y^2+6=13y$$

$$6y^2-13y+6=0$$

$$6y^2-9y-4y+6=0$$

$$3y(2y-3)-2(2y-3)=0$$

$$(3y-2)(2y-3)=0$$

Either $3y-2=0$ or $2y-3=0$

$3y=2$ $\qquad$ $2y=3$

$y=\frac{2}{3}$ $\qquad$ $y=\frac{3}{2}$

Put $y=\frac{2}{3}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$

$$\frac{4x+1}{4x-1}=\frac{2}{3}$$

$$3(4x+1)=2(4x-1)$$

$$12x+3=8x-2$$

$$12x-8x=-2-3$$

$$4x=-5$$

$$x=-\frac{5}{4}$$

Put $y=\frac{3}{2}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$

$$\frac{4x+1}{4x-1}=\frac{3}{2}$$

$$3(4x-1)=2(4x+1)$$

$$12x-3=8x+2$$

$$12x-8x=-2+3$$

$$4x=5$$

$$x=\frac{5}{4}$$

Thus, solution set = $\left\{\pm\frac{5}{4}\right\}$

9. $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution:

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12} \quad \text{———— (i)}$$

Let $\frac{x-a}{x+a}=y$

So eq (i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$

Multiplying both sides by 12y, we get

$$12y^2-12=7y$$

$$12y^2-7y-12=0$$

$$12y^2-16y+9y-12=0$$

$$4y(3y-4)+3(3y-4)=0$$

Either $4y+3=0$ or $3y-4=0$

$4y=-3$ $\quad$ $3y=4$

$y=-\frac{3}{4}$ $\quad$ $y=\frac{4}{3}$

Put $=y-\frac{3}{4}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=-\frac{3}{4}$$

$$4(x-a)=-3(x+a)$$

$$4x-4a=-3x-3a$$

$$4x+3x=4a-3a$$

$$7x=a$$

$$x=\frac{a}{7}$$

Put $=y-\frac{4}{3}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=\frac{4}{3}$$

$$4(x+a)=3(x-a)$$

$$4x+4a=3x-3a$$

$$4x-3x=-4a-3a$$

$$x=-7a$$

Thus, solution set $=\left\{-7a,\frac{a}{7}\right\}$

(10) $x^4-2x^3-2x^2+2x+1=0$

Solution:

$$x^4-2x^3-2x^2+2x+1=0$$

Dividing each term by x^2, we get

$$\frac{x^4}{x^2}-2\frac{x^3}{x^2}-2\frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{2}{x}\right)-2=0 \_\_\_\_(i)$$

Let $x-\frac{1}{x}=y$

$$\left(x-\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}-2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

So eq. (i) becomes

$$y^2+2-2y-2=0$$

$$y^2-2y=0$$

$$y(y-2)=0$$

Either $y=0$ or $y-2=0, \Rightarrow y=2$

Put $y=0$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$

$$x-\frac{1}{x}=0$$

$$\Rightarrow x^2-1=0$$

$$x^2=1$$

$$x=\pm 1$$

Put $y=2$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$

$$x-\frac{1}{x}=2$$

$$\Rightarrow x^2-1=2x$$

$$x^2-2x-1=0$$

$$x = \frac{(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = 1 \pm \sqrt{2}$$

Thus, solution set = $\{\pm 1, 1 \pm \sqrt{2}\}$

(11) $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution:

$$2x^4 + x^3 - 6x^2 + x + 2 = 0$$

Dividing both sides by x^2, we get

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{2}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \text{\_\_\_\_\_\_ (i)}$$

Let $x + \frac{1}{x} = y$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So eq. (i) becomes

$$2(y^2-2)+y-6=0$$
$$2y^2-4+y-6=0$$
$$2y^2+y-10=0$$
$$2y^2+5y-4y-10=0$$
$$y(2y+5)-2(2y+5)=0$$
$$(y-2)(2y+5)=0$$

Either $y-2=0$ or $2y+5=0$

$y=2$ $2y=-5$

$$y=-\frac{5}{2}$$

Put $y=2$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=2$$
$$\Rightarrow x^1+1=2x$$
$$x^2-2x+1=0$$
$$(x-2)^2=0$$
$$\Rightarrow x-1=0$$
$$x=1$$

Put $y=-\frac{5}{2}$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=-\frac{5}{2}$$
$$\Rightarrow x^2+1=2=-5x$$
$$2x^2+5x+2=0$$
$$2x^2+4x+x+2=0$$
$$2x(x+2)+1(x+2)=0$$

Either $2x+1=0$ $x+2=0$

$2x=-1$ $x=-2$

$$x=-\frac{1}{2}$$

Thus, solution set = $\left\{1,-2,-\frac{1}{2}\right\}$

(12) $4.2^{2x+1}-9.2^x+1=0$

Solution:

$$4.2^{2x+1}-9.2^x+1=0$$
$$4.2^{2x}.2^1-9.2^x+1=0 \quad \text{\_\_\_\_\_(i)}$$

Let $2^x=y$ Then $2^{2x}=y^2$

So eq. (i) becomes.

$$4.y^2.2-9.y+1=0$$
$$8y^2-9y+1=0$$
$$8y^2-8y-y+1=0$$
$$8y(y-1)-1(y-1)=0$$
$$(8y-1)(y-1)=0$$

Either $8y-1=0$ or $y-1=0$

$8y=1$ $\quad y=1$

$$y=\frac{1}{8}$$

Put $y=\frac{1}{8}$ in $2^x=y$; we get

$$2^x=y$$
$$2^x=\frac{1}{8}$$
$$2^x=\frac{1}{2^3}$$
$$2^x=2^3$$
$$\Rightarrow x=-3$$

Put $y=1$ in $2^x=y$, we get

$$2^x=y$$
$$2^x=1$$
$$2^x=2=0$$
$$\Rightarrow x=0$$

Thus, solution set = {−3, 0}

13. $3^{2x+2}=12.3^x-3$

Solution:

$$3^{2x+2}=12.3^x-3$$
$$3^{2x}.3^2-12.3^x+3=0 \quad \text{\_\_\_\_\_\_(i)}$$

Let $3^x=y$. Then $3^{2x}=y^2$

So eq. (i) becomes

$$y^2.9-12y+3=0$$
$$9y^2-12y+3=0$$
$$9y^2-9y-3y+3=0$$
$$9y(y-1)-3(y-1)=0$$
$$(9y-3)(y-1)=0$$

Either $9y-3=0$ or $y-1=0$

$9y=3$ $\quad y=1$

$$y=\frac{3}{9}$$
$$y=\frac{1}{3}$$

Put $y = \frac{1}{3}$ in $3^x = y$, we get

$$3^x = y$$
$$3^x = \frac{1}{3}$$
$$3^x = 3^{-1}$$
$$\Rightarrow x = -1$$

Put $y = 1$ in $3^x = y$, we get

$$3^x = y$$
$$3^x = 1$$
$$3^x = 3^0$$
$$\Rightarrow x = 0$$

Thus, solution set = $\{-1, 0\}$

(14) $2^x + 64.2^{-x} - 20 = 0$

Solution:

$$2^x + 64.2^{-x} - 20 = 0 \_\_\_\_\_\_ (i)$$

Let $2^x = y$. Then $2^{-x} = \frac{1}{y}$

So eq (i) becomes

$$y - 64.\frac{1}{y} = 20 = 0$$
$$\Rightarrow \quad y^2 - 64 - 20y = 0$$
$$y^2 - 20y - 64 = 0$$
$$y^2 - 16y - 4y - 64 = 0$$
$$y(y-16) - 4(y-16) = 0$$
$$(y-4)(y-16) = 0$$

Either $y - 4 = 0$ or $y - 16 = 0$

$y = 4$ $\quad$ $y = 16$

Put $y = 4$ in $2^x = y$, we get

$$2^x = y$$
$$2^x = 4$$
$$2^x = 2^2$$
$$\Rightarrow \quad x = 2$$

Put $y = 16$ in $2^x = y$, we get

$$2^x = y$$
$$2^x = 16$$
$$2^x = 2^4$$
$$\Rightarrow \quad x = 4$$

Thus, solution set = $\{2, 4\}$

(15) $(x+1)(x+3)(x-5)(x-7) = 192$

Solution:

$$(x+1)(x+3)(x-5)(x-7) = 192$$

As $1 - 5 = 3 - 7$

So $\quad [(x+1)(x-5)][(x+3)(x-7)] = 192$

$$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$$
$$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \_\_\_\_\_ (i)$$

$$(y-5)(y-21)=192$$
$$y^2-21y-5y+105=192$$
$$y^2-26y+105-192=0$$
$$y^2-26y-87=0$$
$$y^2-29y+3y-87=0$$
$$(y+3)(y-29)=0$$

Either $y+3=0$ or $y-29=0$

$y=-3$ $\qquad$ $y=29$

Put $y=-3$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=-3$$
$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ or $x-3=0$

$x=1$ $\qquad$ $x=3$

Put $y=29$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=29$$
$$x^2-4x-29=0$$

Here $a=1, b=-4, c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{2\left(2\pm\sqrt{33}\right)}{2}$$
$$x=2\pm\sqrt{33}$$

Thus, solution set $=\left\{1,3,2\pm\sqrt{33}\right\}$

(16) (x – 1) (x – 2) (x – 8) (x + 5) 360 = 0

Solution:

$(x-1)(x-2)(x-8)(x+5)\,360 = 0$

As $-1-2 = -8+5$

$-3 = -3$

So $\left[(x-1)(x-2)\right]\left[(x-8)(x+5)\right]+360=0$

$\left[x^2-2x-x+2\right]\left[x^2+5x-8x-40\right]+360=0$

$\left(x^2-3x+2\right)\left(x^2-3x-40\right)+360=0$ ____ (i)

Let $x^2-3x=y$

So eq (i) become

$$(y+2)(y-40)+360=0$$
$$y^2-40y+2y-80+360=0$$
$$y^2-38y+280=0$$
$$y^2-28y-10y+280=0$$
$$y(y-28)-10(y-28)=0$$
$$(y-10)(y-28)=0$$
$$(y-10)(y-28)=0$$

Either $y-10=0$ or $y-28=0$

$y=10$ $y=28$

Put y = 10 in $x^2 - 3x = y$, we get

$$x^2-3x=y$$
$$x^2-3x=10$$
$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$(x+2)(x-5)=0$$

Either $x+2=0$ or $x-5=0$

$x=-2$ $x=5$

Put y = 10 in $x^2 - 3x = y$, we get

$$x^2-3x=y$$
$$x^2-3x=28$$
$$x^2-3x-28=0$$
$$x^2-7x+4x-28=0$$
$$x(x-7)+4(x-7)=0$$
$$(x+4)(x-7)=0$$

Either $x+4=0$ or $x-7=0$

$x=-4$ $x=7$

Thus, solution set = {–4, –2, 5,7}

Radical equations:

An equation involving expression under the radical sign is called a radical equation.

e.g., $\sqrt{x+3} = x+1$ and $\sqrt{x-1}=\sqrt{x-2}+1$

SOLVED EXERCISE 1.4

Solve the following equations.

(1) $2x+5=\sqrt{7x+16}$

Solution:

$2x+5=\sqrt{7x+16}$ _____ (i)

Squaring both sides, we get

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$

$$4x^2+20x+25=7x+16$$

$$4x^2+20x-7x+25-16=0$$

$$4x^2+13x+9=0$$

$$4x^2+9x+4x+9=0$$

$$x(4x+9)+1(4x+9)=0$$

$$(x+1)(4x+9)=0$$

Either $x+1=0$ or $4x+9=0$

$x=-1$ $4x=-9$

$x=-\frac{9}{4}$

Check:

Put $x=-1$ in eq. (i), we get

$2(-1)+5=\sqrt{7(-1)+16} \Rightarrow -2+5=\sqrt{-7+16}$

$3=\sqrt{9} \Rightarrow 3=3$ (which is true)

Put $x=-\frac{9}{4}$ in eq. (i), we get

$2\left(-\frac{9}{4}\right)+5=\sqrt{7\left(-\frac{9}{4}\right)+16} \Rightarrow -\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$

$\frac{1}{2}=\sqrt{\frac{1}{4}} \Rightarrow \frac{1}{2}=\frac{1}{2}$ (Which is true)

Thus, solution set $=\left\{-1,-\frac{9}{4}\right\}$

2) $\sqrt{x+3}=3x-1$

Solution:

$\sqrt{x+3}=3x-1$ _____ (i)

Squaring both sides, we get

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$
$$x+3=9x^2-6x+1$$
$$9x^2-6x+1-x-3=0$$
$$9x^2-7x-2=0$$
$$9x^2-9x+2x-2=0$$
$$9x(x-1)+2(x-1)=0$$
$$(9x+2)(x-1)=0$$

Either $9x+2=0$ or $x-1=0$

$9x=-2$ $\qquad$ $x=1$

$$x=-\frac{2}{9}$$

Check:

Put $x=-\frac{2}{9}$ in eq (i), we get

$\sqrt{-\frac{2}{9}+3}=3\left(-\frac{2}{9}\right)-1$ $\Rightarrow$ $\sqrt{\frac{25}{9}}=-\frac{2}{3}-1$

$\sqrt{\frac{25}{9}}=-\frac{5}{3}$ $\Rightarrow$ $\frac{5}{3}\neq-\frac{5}{3}$ (which is not true)

Put $x=1$ in eq. (i), we get

$\sqrt{1+3}=3(1)-1$ $\Rightarrow$ $\sqrt{4}=3-1$

$2=2$ (Which is true)

Thus, solution set $=\{1\}$

(3) $4x=\sqrt{13x+14}-3$

Solution:

$4x=\sqrt{13x+14}-3$ ______ (i)

$4x+3=\sqrt{13x+14}$

Squering both sides, we get

$$(4x+3)^2=\left(\sqrt{13x+14}\right)^2$$
$$16x^2+24x+9=13x+14$$
$$16x^2+24x-13x+9-14=0$$
$$16x^2+11x-5=0$$
$$16x^2+16x-5x-5=0$$
$$16x(x+1)-5(x+1)=0$$
$$(16x-5)(x+1)=0$$

Either $16x-5=0$ or $x+1=0$

$16x = 5$ $x = -1$

$x = \frac{5}{16}$

Check:

Put $x = \frac{5}{16}$ in eq. (i), we get

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3 \Rightarrow \frac{5}{4} = \sqrt{\frac{65}{16} + 14} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3 \Rightarrow \frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{5}{4}$ (Which is true)

Put $x = -1$ in eq, (i), we get

$4(-1) = \sqrt{13(-1) + 14} - 3 \Rightarrow -4 = \sqrt{-13 + 14} - 3$

$-4 = \sqrt{1} - 3 \Rightarrow -4 = 1 - 3$

$-4 \neq -2$ (Which is not true)

Thus, solution set = $\left\{\frac{5}{16}\right\}$

4. $\sqrt{3x + 100} - x = 4$

Solution:

$\sqrt{3x + 100} - x = 4$ ______ (i)

$\sqrt{3x + 100} = x + 4$

Squaring both sides

$\left(\sqrt{3x + 100}\right)^2 = (x + 4)^2$

$3x + 100 = x^2 + 8x + 16$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x + 12) - 7(x + 12) = 0$

Either $x - 7 = 0$ or $x + 12 = 0$

$x = 7$ $x = -12$

Check:

Put $x = 7$ in eq. (i), we get

$\sqrt{3(7) + 100} - 7 = 4 \Rightarrow \sqrt{21 + 100} - 7 = 4$

$\sqrt{121} - 7 = 4 \Rightarrow 11 - 7 = 4$

$4 = 4$ (Whih is true)

Put $x = -12$ in eq. (i), we get

$\sqrt{3(-12)+100}-(-12)=4 \quad \Rightarrow \quad \sqrt{-36+100}+12=4$

$\sqrt{64}=12=4 \quad \Rightarrow \quad 8+12=4$

$20 \neq 4$ (Which is nt true)

Thus, Solution set = {7}

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ ______ (i)

Squaring both sides, we get

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)}=x+60$

$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$

$2x+26+2\sqrt{x^2+26x+105}=x+60$

$2\sqrt{x^2+26x+105}=x+60-2x-26$

$2\sqrt{x^2+26x+105}=-x+34$

$2\sqrt{x^2+26+105}=-(x-34)$

Squaring both sides, we get

$\left(2\sqrt{x^2+26+105}\right)^2=\left[-(x-34)\right]^2$

$4\left(x^2+26x+105\right)=x^2-68x+1156$

$4x^2+104x+420=x^2-68x+1156$

$4x^2-x^2+104x+68x+420-1156=0$

$3x^2+172x-736=0$

Here a = 3, b = 172, c = – 736

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-172 \pm \sqrt{(172)^2 - 4(3)(-736)}}{2(3)}$$

$$x = \frac{-172 \pm \sqrt{29584 + 8832}}{6}$$

$$x = \frac{-172 \pm \sqrt{38416}}{6}$$

$$x = \frac{-172 - 196}{6} \quad \text{or} \quad x = \frac{-172 + 196}{6}$$

$$x = -\frac{368}{6} \qquad x = \frac{24}{6}$$

$$x - \frac{184}{3} \qquad x = 4$$

Check:

$x = -\frac{184}{3}$ in eq. (i), we get

$$\sqrt{-\frac{184}{3} + 5} + \sqrt{\frac{-184}{3} + 21} = \sqrt{-\frac{184}{3} + 60}$$

$$\sqrt{-\frac{169}{3}} + \sqrt{-\frac{121}{3}} = \sqrt{-\frac{4}{3}} \quad \text{(Which is not true)}$$

Put $x = 4$ in eq. (i), we get

$$\sqrt{4+5} + \sqrt{4+21} = \sqrt{4+60}$$

$$\sqrt{9} + \sqrt{25} = \sqrt{64}$$

$$3 + 5 = 8$$

$$8 = 8 \quad \text{(Which is true)}$$

Thus, solution set = $\{8\}$

(6) $\sqrt{x-1} + \sqrt{x-2} + \sqrt{x+6}$

Solution:

$$\sqrt{x-1} + \sqrt{x-2} + \sqrt{x+6} \quad \text{\_\_\_\_\_\_\_ (i)}$$

Squaring both sides, we get

$$\left(\sqrt{x+1} + \sqrt{x-2}\right)^2 = \left(\sqrt{x+6}\right)^2$$

$$(x+1) + (x-2) + 2\sqrt{(x+1)(x-2)} = x + 6$$

$$x + 1 + x - 2 + 2\sqrt{x^2 - x - 2} = x + 6$$

$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2\sqrt{x^2-x-2}=x+6-2x+1$$
$$2\sqrt{x^2-x-2}=-x+7$$
$$2\sqrt{x^2-x-2}=-(x-7)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$$
$$4\left(x^2-x-2\right)=x^2-14x+49$$
$$4x^2-4x-8=x^2-14x+49$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$

Here a = 3, b = 10, c = – 57

$$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$$
$$x=\frac{-10\pm\sqrt{100+684}}{6}$$
$$x=\frac{-10\pm\sqrt{784}}{6}$$
$$x=\frac{-10\pm 28}{6}$$
$$x=\frac{-10-28}{6} \quad \text{or} \quad x=\frac{-10+28}{6}$$
$$x=\frac{-38}{6} \qquad x=\frac{18}{6}$$
$$x=-\frac{19}{3} \qquad x=3$$

Check:

Put $x=-\frac{19}{3}$ in eq. (i), we get

$$\sqrt{-\frac{19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{-\frac{19}{3}+6}$$
$$\sqrt{\frac{-16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}} \quad \text{(Which is not true)}$$

Put x = 3 in eg. (i), we get

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$
$$2+1=3$$
$$3=3 \text{ (Which is true)}$$

Thus, solution set = $\{3\}$

(7) $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

Solution:

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x} \qquad \text{\_\_\_\_\_\_\_ (i)}$$

Squaring both sides, we get

$$\left(\sqrt{11-x}+\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$
$$(11-x)+(6-x)+2\sqrt{(11-x)(6-x)}=27-x$$
$$11-x+6-x+2\sqrt{(11-x)(6-x)}=27-x$$
$$17-2x+2\sqrt{x^2-17x+66}=27-x$$
$$2\sqrt{x^2-17x+66}=27-x-17+2x$$
$$2\sqrt{x^2-17x+66}=10+x$$

Squaring both sides, we get

$$\left(2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$$
$$4\left(x^1-17x+66\right)=100+20x+x^2$$
$$4x^2-68x+264=x^2+20x+100$$
$$4x^2-x^2-68x+20x+264-100=0$$
$$3x^2-88x+164=0$$

Here $a=3, b=-88, c=164$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-88) \pm \sqrt{(-88)^2 - 4(3)(164)}}{2(3)}$$

$$x = \frac{88 \pm \sqrt{7744 - 1968}}{6}$$

$$x = \frac{88 \pm \sqrt{5776}}{6}$$

$$x = \frac{88 \pm 76}{6}$$

$$x = \frac{88 - 76}{6}, \qquad x = \frac{88 + 76}{6}$$

$$x = \frac{12}{6}, \qquad x = \frac{164}{6}$$

$$x = 2 \qquad x = \frac{82}{3}$$

Check:

Put $x = 2$ in eq. (i), we get

$$\sqrt{11-2} + \sqrt{6-2} = \sqrt{27-2}$$

$$\sqrt{9} + \sqrt{4} = \sqrt{25}$$

$$3 + 2 = 5 \quad \Rightarrow \quad 5 = 5 \text{ (Which is true)}$$

Put $x = \frac{82}{3}$ in eq. (i), we get

$$\sqrt{11 - \frac{82}{3}} + \sqrt{6 - \frac{82}{3}} = \sqrt{27 - \frac{82}{3}}$$

$$\sqrt{-\frac{49}{3}} + \sqrt{-\frac{64}{3}} = \sqrt{-\frac{1}{3}} \text{ (Which is not true)}$$

Thus, Solution set = {2}

(8) $\sqrt{4a + x} - \sqrt{a - x} = \sqrt{a}$

Solution:

$$\sqrt{4a + x} - \sqrt{a - x} = \sqrt{a}$$

Squaring both sides, we get

$$\left(\sqrt{4a + x} - \sqrt{a - x}\right)^2 = \left(\sqrt{a}\right)^2$$

$$(4a + x) - (a - x) - 2\sqrt{(4a + x)(a - x)} = a$$

$$4a+x-a+x-2\sqrt{4a^2-3ax-x^2}=a$$

$$3a+2x-2\sqrt{4a^2-3ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-3a-2x$$

$$-2\sqrt{4a^2-3ax-x^2}=-2a-2x$$

$$-2\sqrt{4a^2-3ax-x^2}=-2(a+x)$$

$\Rightarrow$ $\sqrt{4a^2-3ax-x^2}=(a+x)$

Squaring both sides, we get

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(a+x)^2$$

$$4a^2-3ax-x^2=a^2+x^2+2ax$$

$$-x^2-x^2-3ax-2ax+4a^2-a^2=0$$

$$-2x^2-5ax+3a^2=0$$

$$-\left(2x^2+5ax-3a^2\right)=0$$

$\Rightarrow$ $2x^2+5ax-3a^2=0$

$$2x^2+6ax-ax-3a^2=0$$

$$2x(x+3a)-a(x+3a)=0$$

$$(2x-a)(x+3a)=0$$

Either $2x-a=0$ or $x+3a=0$

$2x=a$ $\quad$ $x=-3a$

$x=\frac{a}{2}$

Thus, Solution set $=\left\{-3a,\frac{a}{2}\right\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \qquad \text{\_\_\_\_\_\_ (i)}$$

Let $x^2+x=y$

So eq. (i) becomes

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides, we get

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$$

$$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$$

$$y+1+y-1-2\sqrt{y^1-1}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$-2\sqrt{y^2-1}=1-2y$$

Squaring both sides, we get

$$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$$

$$4\left(y^2-1\right)=1-4y+4y+4y^2$$

$$4y^2-4=1-4y+4y^2$$

$$4y^2-4-1+4y-4y^2=0$$

$$4y-5=0$$

$$y=\frac{5}{4}$$

Put $y=\frac{5}{4}$ in $x^2+x=y$, we get

$$x^2+x=\frac{5}{4}$$

$\Rightarrow$ $4x^2+4x=5$

$$4x^2+4x-5=0$$

Here $a=4,\ b=4,\ c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{88\pm 76}{6}$$

$$x=\frac{-4\pm 4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \quad \text{\_\_\_\_\_\_(i)}$$

Let $x^2 + 3x = y$

So eq. (i) becomes

$$\sqrt{y+8} - \sqrt{y+2} = 3$$

Squaring both sides, we get

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = 9$$

$$(y+8) + (y+2) + 2\sqrt{(y+8)(y+2)} = 9$$

$$y + 8 + y + 2 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2\sqrt{y^2 + 10y + 16} = 9 - 2y - 10$$

$$2\sqrt{y^2 + 10y + 16} = -2y - 1$$

$$2\sqrt{y^2 + 10y + 16} = -(2y + 1)$$

Squaring both sides, we get

$$\left(2\sqrt{y^2 + 10y + 16}\right)^2 = \left[-(2y+1)\right]^2$$

$$4\left(y^2 + 10y + 16\right) = 4y^2 + 4y + 1$$

$$4y^2 + 40y + 64 = 4y^2 + 4y + 1$$

$$4y^2 - 4y^2 + 40y - 4y + 64 - 1 = 0$$

$$36y + 63 = 0$$

$$36y = -63$$

$$y = -\frac{63}{36}$$

Put $y = -\frac{63}{36}$ in $x^2 + 3x = y$, we get

$$x^2 + 3x = -\frac{53}{36}$$

$\Rightarrow$ $36x^2 + 108x = -63$

$$36x^2 + 108x + 63 = 0$$

Here $a = 36$, $b = 108$, $c = 63$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x=\frac{108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$x=\frac{-108\pm\sqrt{11664+9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{108\pm 36\sqrt{2}}{72}$$

$$x=\frac{36\left(-3\pm\sqrt{2}\right)}{72}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

Thus, Solution set = $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

11) $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5 \quad \_\_\_\_ (i)$$

Let $x^2+3x=y$

So eq. (i) becomes

$$\sqrt{y+9}+\sqrt{y+1}=5$$

Squaring both sides, we get

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=25$$

$$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$$

$$y+9+y+4+2\sqrt{y^2+13+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=25-2y+12$$

$$2\sqrt{y^2+13y+36}=25-2(y-6)$$

$\Rightarrow$ $$\sqrt{y^2+13y+36}=-(y-6)$$

Squaring both sides, we get

$$\left(\sqrt{y^2+13y+36}\right)^2\left[-(y-6)\right]^2$$

$$y^2 + 13y + 36 = y^2 - 12y + 36$$
$$y^2 - y^2 + 13y + 12y + 36 - 36 = 0$$
$$25y = 0$$
$$\Rightarrow \quad y = 0$$

Put $y = 0$ in $x^2 + x^2 + y$, we get

$$x^2 + 3x = y$$
$$x^2 + 3x = 0$$
$$x^2 + 3x = 0$$
$$x(x + 3) = 0$$

Either $x = 0$ or $x + 3 = 0$

Thus, solution set = $\{-3, 0\}$

SOLVED MISCELLANEOUS EXERCISE - 1

Q1. Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is:

(a) $bx + c = 0,\ b \neq 0$ (b) $ax^2 + bx + c = 0,\ a \neq 0$

(c) $ax^2 = bx,\ a \neq 0$ (d) $ax^2 = 0,\ a \neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

(iii) The number of methods to solve a quadrate equation is:

(a) 1 (b) 2 (c) 3 (d) 4

(iv) The quadratic formula is:

(a) $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

(c) $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ (d) $\dfrac{b \pm \sqrt{b^2 + 4ac}}{2a}$

(v) Two linear factors of $x^2 - 15x + 56$ are:

(a) $(x - 7)$ and $(x + 8)$ (b) $(x + 7)$ and $(x - 8)$

(c) $(x - 7)$ and $(x - 8)$ (d) $(x + 7)$ and $(x + 8)$

(vi) An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an

(a) Exponential equation (b) Reciprocal equation

(c) Radical equation (d) None of these

(vii) An equation 6fthetype $3^x + 3^{2x} + 6 = 0$ is a/an:

(a) Exponential equation (b) Radical equation

(c) Reciprocal equation (d) None of these

(viii) The solution set of equation $4x^2 - 16 = 0$ is:

(a) $\{\pm 4\}$ (b) $\{4\}$ (c) $\{\pm 2\}$ (d) ± 2

(ix) An equation of the form $2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$ is called a/an

(a) Reciprocal equation (b) Radical equation

(c) Exponential equation (d) None of these

Answers:

(i)	b	**(ii)**	c	**(iii)**	c	**(iv)**	a	**(v)**	c
(vi)	b	**(vii)**	a	**(viii)**	c	**(ix)**	a		

Q2. Write short answers of the following questions.

(i) Solve $x^2 + 2x - 2 = 0$

Ans:

$x^2 + 2x - 2 = 0$

Compare it with

$ax^2 + bx + c = 0$

Here a = 2, b = 2, c = 2

By using quadratic equation, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{4 + 16}}{4}$$

$$x = \frac{-2 \pm \sqrt{20}}{4} = \frac{-2 \pm 2\sqrt{5}}{4}$$

$$x = \frac{2(-1 \pm \sqrt{5})}{4} = -1 \pm \sqrt{5}$$

Thus, the solution set = $\{-1 \pm \sqrt{5}\}$

(ii) Solve by factorization $5x^2 = 15x$

Ans:

$5x^2 = 15x$

$5x^2 - 15x = 0$

$5x(x - 3) = 0$

Either $5x = 0$ or $x - 3 = 0$

$\Rightarrow$ $x = 0$ $x = 0$

Thus, the solution set = $\{0, 3\}$

(iii) Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

Ans:

$$\frac{1}{x+4}+\frac{1}{x-4}=3 \Rightarrow \qquad \frac{x-4+x+4}{(x+4)(x-4)}=3$$

$$\frac{2x}{x^2-16}=3 \Rightarrow \qquad 3(x^2-16)=2x$$

$$3x^2-48-2x=0$$

$$3x^2-2x-48=0$$

(iv) Write the names of the methods for solving a quadratic equation.

Ans: Solution of quadratic equations:

To find solution set of a quadratic equation, following methods are used:

(i) Factorization (ii) Completing square

(v) Solve $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Ans:

$$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$$

Taking square root on both sides, we get

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$

$$2x=\frac{1}{2}\pm\frac{3}{2}$$

Either $2x=\frac{1}{2}+\frac{3}{2}$ or $2x=\frac{1}{2}-\frac{3}{2}$

$2x=2$ $\qquad$ $2x=-1$

$\Rightarrow$ $x=1$ $\qquad$ $x=-\frac{1}{2}$

Thus, the solution set= $\left\{-\frac{1}{2},1\right\}$

(vi) Solve $\sqrt{3x+18}=x$

Ans:

$$\sqrt{3x+18}=x$$

Taking square on both sides, we get

$$3x+18=x^2$$

$$x^2-3x-18=0$$

$$x^2 - 6x + 3x - 18 = 0$$
$$x(x-6) + 3(x-6) = 0$$
$$(x-6)(x+3) = 0$$

Either $x - 6 = 0$ or $x + 3 = 0$

$x = 6$ $\quad$ $x = -3$

Thus, the solution set = $\{-3, 6\}$

(vii) Define quadratic equation.

Ans: Quadratic Equation:

An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a **quadratic equation** or an equation of the second degree.

(viii) Define exponential equation.

Ans: Exponential equations:

In exponential equations, variable occurs in exponent.

For example, $5^{1+x} + 5x^{1-x} = 26$.

(ix) Define reciprocal equation.

Ans: Reciprocal equations of the type:

$$\mathbf{a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) = c = 0} \text{ or } ax^4 + bx^3 + cx^2 + bx + a = 0$$

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Replacing x by $\frac{1}{x}$ in $ax^4 - bx^3 + cx^2\ bx + a = 0$, we have

$$a\left(\frac{1}{x}\right)^4 - b\left(\frac{1}{x}\right)^3 + c\left(\frac{1}{x}\right)^2 - b\left(\frac{1}{x}\right) + a = 0 \text{ which is simplified as}$$

$a - bx + cx^2 - by^3 + ax^4 = 0$. We get the same equation.

Thus $ax^4 - bx^3 + cx^2 - bx + a = 0$ is a reciprocal equation.

(x) Define radical equation.

Ans: Radical equations:

An equation involving expression under the radical sign is called a **radical equation**.

e.g., $\sqrt{x+3} = x + 1$ and $\sqrt{x-1} = \sqrt{x-2} + 1$

Q3. Fill in the blanks:

(i) The standard form of the quadratic equation is_________.

(ii) The number of methods to solve a quadratic equation are_________.

(iii) The name of the method to derive a quadratic formula is_________.

(iv) The solution of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is_________.

(v) The solution set of $25x^2 - 1 = 0$ is _________.

(vi) An equation of the form $2^{2x} - 3.2^{x} + 5 = 0$ is called a/an_________ equation.

(vii) The solution set of the equation $x^2 - 9 = 0$ is _________.

(viii) An equation of the type $x^4 + x^3 + x^2 + x + 1 = 0$ called a/an

(ix) A root of an equation, which do not satisfy the equation is called_________ root.

(x) An equation involving impression of the variable under_________ is called radical.

Answer

(i)	$ax^2+bx+c=0$	**(ii)**	3	**(iii)**	Completing Square
(iv)	$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**(v)**	$\left\{\pm \dfrac{1}{5}\right\}$	**(vi)**	Exponential
(vii)	$\{\pm 3\}$	**(viii)**	Reciprocal	**(ix)**	Extraneous
(x)	Radical sign				

SUMMARY

- An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.
- A second degree equation in one variable x, $ax^2 + bx + c = 0$
- Where $a \neq 0$ and a, b, c are real numbers, is called the general or standard form of a quadratic equation.
- An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\dfrac{1}{x}$.
- In exponential equations, variables occur in exponents.
- An equation involving expression under the radical sign is called a radical equation.
- Quadratic formula for $ox^2 + bx + c = 0$, $a \neq 0$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Any quadratic equation is solved by the following three methods.
  (i) Factorization (ii) Completing square (iii) Quadratic formula

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